The Planet Temperature Calculator

http://www.astro.indiana.edu/~gsimonel/temperature1.html

Many astrobiologists think that in order for a planet to support human life, water must be able to exist as a liquid on it. That means that somewhere on the planet the temperature, assuming reasonable pressure, must be between 273 and 373 degrees Kelvin. It is sometimes helpful to think of "habitable zones" for planets. A habitable zone is an area surrounding a star where the surface temperature of a planet in that zone is between $273°K$ and $373°K$.

The Planet Temperature Calculator can evaluate whether a specific planet is in a habitable zone. The program requires the user to input 4 parameters that affect the surface temperature of a planet. It then computes the surface temperature for those parameters. The 4 parameters are: mass of the nearest star, distance from the nearest star, bond albedo and greenhouse effect.

Mass

The amount of energy given off by a star is directly related to its mass. For this program we are assuming that the star is a main sequence star, where its energy output is relatively easy to predict. A main sequence star is a star that is in the main part of its life cycle—it is not just becoming a star, nor is it about to use up all its nuclear fuel and burn out or explode. The equation for the amount of energy emitted is: $L = 3.846 * 1033 * M^3 e/s$, where L is the luminosity in ergs per second and M is the mass of the star in solar masses. (One solar mass equals 2.0×10^{33} grams.) A solar mass is a comparison of a star's mass with the mass of our Sun. Our Sun has a solar mass of 1. For this program the solar mass must be between 0.08 and 100. The low cutoff point of 0.08 was chosen because below that point a star does not have enough mass to initiate fusion. Stars can and do have solar masses above 100, but above that they go through their life cycle very quickly, in less than ten million years, so they are likely to destroy any planet surrounding them before life could have a chance to become established.

Distance

Distance is entered in Astronomical Units (AU). An AU is the average distance from the Earth to the Sun, approximately 93 million miles. For this program the distance is calculated in cm, using the formula $D = AU * 1.496 * 10^{33} cm$. In this program distance is restricted to between 0.1 AU and 100 AU.

Bond Albedo

Bond albedo (BA) is the average percent of the energy reaching a planet's surface that is reflected away from the planet; it represents energy received that does not contribute to heating the planet. In this program bond albedo is entered as a number between 0 and 100, which is then converted to a percent by the program $(A = BA/100)$. The number is a constant—there are no units associated with it, and it is represented by A.

Effective Temperature

Effective temperature is the amount of heat available to heat a planet. Once mass (M), distance (D) and bond albedo (A) are entered, effective temperature can be calculated using the general equation $Rate_{IN} = Rate_{OUT}$. $Rate_{IN}$ is derived from the formula:

$$Rate_{IN} = \left(\frac{L}{4\pi D^2}\right) * \pi R^2 * (1 - A).$$

$\left(\frac{L}{4\pi D^2}\right)$ is “flux” in $erg/cm^2/sec$, πR^2 is the cross section of the planet, and *(1-A)* is the fraction of starlight absorbed. $Rate_{OUT}$ is calculated as $(4\pi R^2)(\sigma T^4)$, where $(4\pi R^2)$ is the area of the planet and (σT^4) is the formula for hot “black” body radiation. σ is the Stefan-Boltzman constant, equal to $5.6703 * 10^{-5} watts/cm^2/T^4$, and *T* is temperature in degrees Kelvin. By canceling out R^2 and solving for *T* we get the following formula for effective temperature:

$$T_{eff} = \left[\frac{(1-A)*L}{16\pi\sigma}\right]^{1/4} * \frac{1}{\sqrt{D}} .$$

Greenhouse Effect

Determining the exact greenhouse effect is beyond the scope of this project. Since this program is designed to be used by middle grade students, it was decided to accept a simple number to represent overall greenhouse effect. We decided that we would call the Earth’s greenhouse effect 1. No greenhouse effect would equal 0. Working backwards using the values for Venus yielded a greenhouse effect of around 200 for the planet. There is no theoretical upper limit for the effect, but since Venus’s greenhouse effect is about 200 times as strong as Earth’s using this formula, we decided to set an upper limit of 500.

We were able to come up with a value for greenhouse effect by starting with the formula:

$$T^4_{ground} = T^4_{eff} * \left(1 + {}^3/_4\, t_{ir}\right)$$

where t_{ir} equals the greenhouse effect. Solving for t_{ir} yields:

$$t_{ir} = {}^4/_3 * \left(\frac{T^4_{ground}}{T^4_{eff}} - 1\right).$$

Substituting known values for Earth yields:

$$t_{ir} = {}^4/_3 * \left(\frac{288^4_{\circ k}}{263^4_{\circ k}} - 1\right) = 0.584.$$

So 0.584 become the greenhouse constant that is multiplied by the greenhouse effect value entered by the user.

Emissivity

Because Earth is not a perfect black body radiator we needed to add a slight corrective factor to T_{eff}. This factor is ε=0.9 and it is multiplied to T_{eff}.

Surface Temperature

Surface temperature can now be calculated with the formula:

$$T_{surface°K} = \sqrt[4]{T_{ground}^{4} \; * \; \varepsilon}.$$

Substituting for T_{ground}^{4} gives:

$$T_{surface°K} = \sqrt[4]{T_{eff}^{4}\left(1 + \left[{}^{3}/_{4} * \; 0.584G\right]\right) * 0.9}.$$

Finally, substituting for T_{eff}^{4} and simplifying yields:

$$T_{surface°K} = \sqrt[4]{\sqrt[4]{\frac{(1-A)*(3.846*10^{33}/M^{3})_{e/s}}{16\pi*5.6705*10^{-5}{}_{watts/cm^{2}/°K^{4}}} * \frac{1}{\sqrt{AU*1.496*10^{13}_{cm}}}} * (1 + 0.438G * 0.9}$$